Reg. No.____________ Name:______________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

Course Code: **MA204**

Course Name: **PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)**

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two questions.

1(a) The probability distribution of a discrete random variable X is given by $P(X=x)=\frac{k}{2^x}$, $x=0,1,2,3,4$. Find (i) the value of k,(ii) the probability that X is even and (iii) E(X). (7)

(b) The probability that an electric component manufactured by a firm is defective is 0.01. If the produced items are sent to the market in packets of 10, find the number of packets containing exactly two defectives and at most two defectives in a consignment of 1000 packets using (i) binomial distributionand (ii) Poisson approximation to binomial distribution. (8)

2(a) Buses arrived at a specified stop at 15 minute intervals starting at 7 AM. A passenger arrives at the stop at random time between 7 AM and 7.30 AM. Find the probability that he waits (i) less than 5 minutes, (ii) at least 12 minutes. (7)

(b) 1000 light bulbs with mean length of life 120 days are installed in a factory. Their length of life is assumed to follow normal distribution with S.D 20 days. How many bulbs will expire in less than 90 days? If it is decided to replace all the bulbs together, what interval should be allowed between replacements if not more than 10% should expire before replacement? (8)

3(a) A communication system sends data in the form of packets of fixed length. Noise in the communication channel may cause a packet to be received incorrectly. If this happens, the packet is retransmitted. Let the probability that a packet is received incorrectly is p. Determine the average number of transmissions that are necessary before a packet is received correctly. (7)

(b) Suppose a new machine is put into operation at time zero. Its life time is an exponential random variable with mean life 12 hours. (i) What is the probability that the machine will work continuously for one day? (ii) Suppose the machine has not failed by the end of the first day, what is the probability that it will work for the whole of the next day? (8)

PART B

Answer any two questions.

4(a) A computer generates 100 random numbers which are uniformly distributed between 0 and 1. Find approximately the probability that their sum is at least 50. (7)

(b) The joint distribution of a two-dimensional random variable (X,Y) is given by p(x,y)=c(2x+3y), x=0,1,2 ; y=1,2,3. Find (i) the value of c (ii) the marginal distributions (iii) Are X and Y independent? (8)

5(a) Prove that the random process X(t) is defined by $X(t)=a\sin(\omega t+\theta)$, where a and ω are constants and θ is a random variable uniformly distributed in $[0,2\pi]$ is a WSS process. (7)

(b) In each of the following examine whether $f(\omega)$ could be the power spectral density(PSD) of a wide sense stationary process. Explain your reasoning.

$$(i)\ f(\omega)=\begin{cases}\dfrac{\sin\omega}{\omega}, & \omega\neq 0\\ 0, & \omega=0\end{cases} \qquad (ii)\ f(\omega)=\begin{cases}\pi, & |\omega|<1\\ 0, & otherwise\end{cases}$$

If $f(\omega)$ is a valid PSD find the corresponding autocorrelationfunction. (8)

6(a) Let X_i are independent random variables taking values -1 and 1 with probability $\frac{1}{2}$.
A random process Z_n is defined as $Z_n=X_1+X_2+...+X_n$, $n=1,2,.....$ Is the process a WSS process? (7)

(b) A pair of random variables X and Y have a joint probability density function given by

$$f(x,y)=\begin{cases}\dfrac{1}{\pi}, & x^2+y^2\le 1\\ 0, & otherwise\end{cases}$$

. Show that X and Y are not independent, but uncorrelated. (8)

PART C

Answer any two questions.

7(a) Obtain the probability distribution of the time between two consecutive occurrences of a Poisson process. (4)

(b) The arrival of patients at a doctor's consulting room is found to follow a Poisson process with an average of one in 5 minutes. The room can accommodate a maximum of 4 persons and if more people come, they wait outside the room. If patients start coming from 8 A.M. onwards, (i) What is the probability that the room is full when thedoctor arrives at 9 A. M.? (ii) If the doctor takes a break from 11A.M. to 11.15 A.M., and a lunch break from 1 P.M to 1.30 P.M. what is the probability that no new patients arrive during both the tea break and lunch break? (8)

(c) The transition probability matrix P of a Markov Chain with three states 1, 2 and 3 is

$$\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

with initial distribution $P(X_0=1)=P(X_0=2)=P(X_0=3) = \frac{1}{3}$. Find $P(X_2=3)$ and $P(X_2=3, X_1=2, X_0=3)$. (8)

8(a) Using Newton-Raphson method, compute a real root of $e^{2x}-x-6=0$ lying between 0 and 1. (4)

(b) Health surveys are conducted in a city every 10 years. The following data gives the number of people (in thousands) having heart diseases as found from the records of the survey.

Year	1961	1971	1981	1991	2001	2011
No. of people	16	19	23	28	34	41

Use Newton's interpolation method to estimate the number of people with heart diseases in the year 2005. (8)

(c) Using Runge-Kutta method of order four, compute y(0.2) given that $\frac{dy}{dx}.=e^x+y$, y(0) = 0. Take step size h = 0.1 (8)

9(a) A meteorologist studying the weather in a region decides to classify each day as simply sunny or cloudy. After analysing several years of weather records, he finds that the day after a sunny day is sunny 80% of the time, and cloudy 20% of the time; and the day after a cloudy day is sunny 60% of the time, and cloudy 40% of the time. (i) If Monday is observed to be cloudy what is the probability that Wednesday is sunny? (ii) If the probability is 0.3 for Monday to be cloudy what is the probability that Wednesday is sunny? (iii) What is the probability for weather to be cloudy on Monday, cloudy again on Tuesday, then sunny on Wednesday and again sunny on Thursday in that order, given that the probability is 0.3 for Monday to be cloudy? (iv) What is the long term probability distribution of weather on any day? (10)

(b) The speed of a moving particle was measured at different points of time. The time t when the first measurement was recorded is taken as t =0. Subsequent speeds at different times are as shown in the following table.

Tim(t) in seconds	0	10	20	30	40	50	60
Velocity(v) in m/sec	35	39	44	50	56	43	40

Using Simpson's one-third method, evaluate the distance travelled by the particle in 60 seconds. (5)

(c) Using Lagrange's interpolation method find the polynomial f(x) which agree with the data f(-1) = 3, f(0) = -4, f(1) = 5 and f(2) = -6 (5)